Appendix I: Diagnostic Statistics

Hansen (1997) suggests a distribution theory for least-squares estimates of the threshold in a similar family of threshold autoregressive (TAR) models as well as a means of forming likelihood ratio statistics in order to gauge the relative performance of TAR models versus standard (linear) autoregressive models. The methodology developed by Hansen was adapted for use with our ATECM model. Specifically, the F-statistic was formed as:

$$F_n = n\left(\frac{\hat{\sigma}_n^{*2} - \hat{\sigma}_n^2(\hat{\gamma})}{\hat{\sigma}_n^2(\hat{\gamma})}\right),$$

where the denominator equals the residual variance of the threshold model and the first term in the numerator equals the residual variance of a linear model; however, as γ (the threshold) is not identified, this F-statistic does not take a χ^2 distribution. Thus, we approximate by using:

$$F_n^* = \sup_{\gamma \in F}\left[n\left(\frac{\tilde{\sigma}_n^{*2} - \tilde{\sigma}_n^{*2}(\gamma)}{\tilde{\sigma}_n^{*2}(\gamma)}\right)\right],$$

where the denominator equals the residual variance of a regression of *n* standard normals on the price margin (M^{12}) and the first term in the numerator equals the residual variance of a regression of *n* standard normals on the price margin less the estimated trade cost (M^{12}-C^{21}). The approximation converges weakly in probability to the null distribution of the F-statistic, so the statistic is bootstraped (with 1000 replications) to approximate the asymptotic distribution, allowing one to calculate:

$$p - value = \frac{count\ if\ F_n^* > F_n}{1000}.$$

For the sampling distribution of the threshold estimate, we form:

$$LR_n(\gamma) = n\left(\frac{\hat{\sigma}_n^2(\gamma) - \hat{\sigma}_n^2(\hat{\gamma})}{\hat{\sigma}_n^2(\hat{\gamma})}\right),$$

for every set of γ used in the grid search. Using the critical values provided by Hansen for $C_\xi(\beta)$, minimum and maximum values of Γ were determined for which:

$$\hat{\Gamma} = \{\gamma : LR_n(\gamma) \le C_\xi(\beta)\}.$$

This range of values for Γ provides confidence intervals from which the standard error was calculated as:

$$\frac{abs(\max\hat{\Gamma} - \min\hat{\Gamma})}{4.3125}.$$

An informal review of the standard errors and p-values strongly suggests the applicability and significance of the ATECM specification.